Estimation of Critical Gaps and Follow-Up Times at Rural Unsignalized Intersections in Germany

ANTJE WEINERT
Ruhr University Bochum, Institute for Traffic Engineering and Transportation
Bochum, Germany

ABSTRACT

New critical gaps and follow-up times were investigated to prepare the improved guideline for unsignalized intersections in Germany. These guidelines will form part of the future *Handbuch zur Bemessung von Straßenverkehrsanlagen* (HBS), the German equivalent of the U.S. *Highway Capacity Manual* (HCM). The studies were based on empirical data collected at unsignalized intersections all over the German territory. Follow-up times were measured directly by observing traffic flow at these intersections; however, the critical gaps could only be derived by measuring those gaps accepted and rejected by drivers. The "maximum likelihood technique" was applied for this purpose. Resulting critical gaps and follow-up times were analyzed to determine their dependence on parameters such as intersection layout, speeds, and volumes. These parameters were tested using the conventional calculation method for intersections without traffic signals. The parameters were then integrated into the capacity estimation diagrams for the new German HBS so that capacity and traffic flow can be determined reliably and appropriately for actual situations.

1. INTRODUCTION

Capacity calculations for unsignalized intersections controlled by Yield or Stop signs are based on gap acceptance theory both in Chapter 10 of the HCM (TRB, 1998) and in the German guidelines (FGSV, 1991). The fundamental parameters are critical gaps and follow-up times. These parameters indicate the dependence of traffic conditions at intersections without traffic signals on drivers' behavior. Each driver waiting in a minor stream has to decide when it is safe to cross the intersection or merge into the conflicting traffic streams. The critical gaps and follow-up times take into account the influence of external parameters, for example, the geometric design of the intersection or type of priority rule, on the drivers' decision-making process.

Capacity formulas based on gap acceptance theory have been improved continously. Kyte et al. (1996) recently tested various capacity models for unsignalized intersections in a large and extensive project. The method developed by Siegloch (1973) and Harders (1968, 1976), which has been corrected by Brilon and Großmann (1989) and is currently used in German standards, turned out to be the method with the most realistic results. However, the values for critical gaps and follow-up times for German conditions that were adopted in this method were determined by Harders in 1976. It was assumed that these parameters have been somewhat affected over the past 20 years as a result of higher traffic volumes, improved car performance, and changed driver behavior. In addition, the accuracy of Harders' values has been questioned by several researchers. For example, Harders' research indicated that the speed of the vehicles on the major road highly influenced the critical gaps. This correlation could not be proven in any other investigation since then.

For these reasons, this study sought to ascertain new critical gaps and follow-up times that are appropriate for actual conditions at unsignalized intersections in Germany. Due to the contractor of the study—the federal DOT—only rural intersections were investigated. Significant influences on the critical gaps and follow-up times were to be analyzed. The aim was to accommodate the capacity calculation method to the results so that capacity and traffic flow at unsignalized intersections could be estimated reliably. These parameters will then be used to establish German guidelines for intersections without traffic signals.

2. METHODOLOGY

2.1 Theoretical Basis for the Determination of Critical Gaps and Follow-Up Times

Considering the most simple form of an unsignalized intersection, an intersection with two streams—one major and one minor stream—(Figure 1), the vehicles in the minor stream can only pass the conflict area when the time gap between the cars in the major stream is long enough. That means they can only enter the conflict area when the time gap between the major vehicles is larger than their critical gap, t_c. Therefore the critical gap t_c is defined as follows:

> The critical gap, t_c, is the minimum time gap between the vehicles of the major stream that is necessary for the vehicles in the minor stream to enter the conflict area.

In addition, several cars of the minor stream can only follow one behind the other within a certain time space, which is called their follow-up time, t_f.

> The follow-up time t_f is the average time gap between two cars of the minor stream being queued and entering the same major stream gap one behind the other.

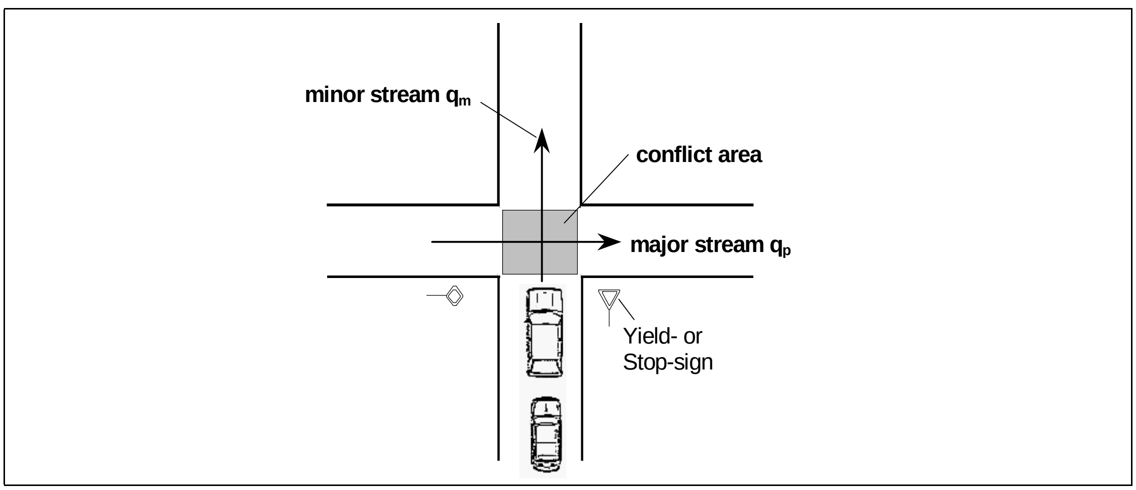

FIGURE 1 Unsignalized intersection with two streams.

The follow-up time can be derived from field observations. However, one cannot measure the critical gap directly. Rather, only the gaps accepted and rejected by the minor stream drivers can be observed in the field. Because the critical gap cannot be measured directly, a procedure is needed that allows the drivers' critical gaps to be estimated using their accepted and rejected gaps.

Several methods to estimate critical gaps are found in the literature. Brilon et al. (1997) tested some of them in simulation studies. They found that the maximum likelihood technique (Troutbeck, 1992) can reproduce the real critical gap of a driver population quite reliably without depending on external parameters. For this reason, the maximum likelihood technique was used to determine critical gaps in the U.S. project by Kyte et al. (1996), as well as for this German empirical investigation.

The maximum likelihood technique is based on the assumption that minor stream drivers behave consistently, meaning that every driver has a certain critical gap that is acceptable. The driver will reject every gap smaller than his or her critical gap and will accept the first gap larger than the critical gap. Under this assumption, a driver's critical gap can be found between his or her largest rejected and the accepted gap. The distribution of the critical gaps within a driver population lies between their largest rejected and their accepted gaps (Figure 2).

First, a function, *F*, which represents the distribution of the critical gaps, is assumed in applying the maximum likelihood procedure. A log-normal function was proposed for this purpose by several researchers (Troutbeck, 1992). The parameters of this function, the mean μ and variance σ^2, are obtained by maximizing the likelihood function. The likelihood function is defined as the probability that the critical gap distribution lies between the observed distribution of the largest rejected gaps and the accepted gaps:

$$L = \prod_{i=1}^{n} \left[F(a_i) - F(r_i)\right] \tag{1}$$

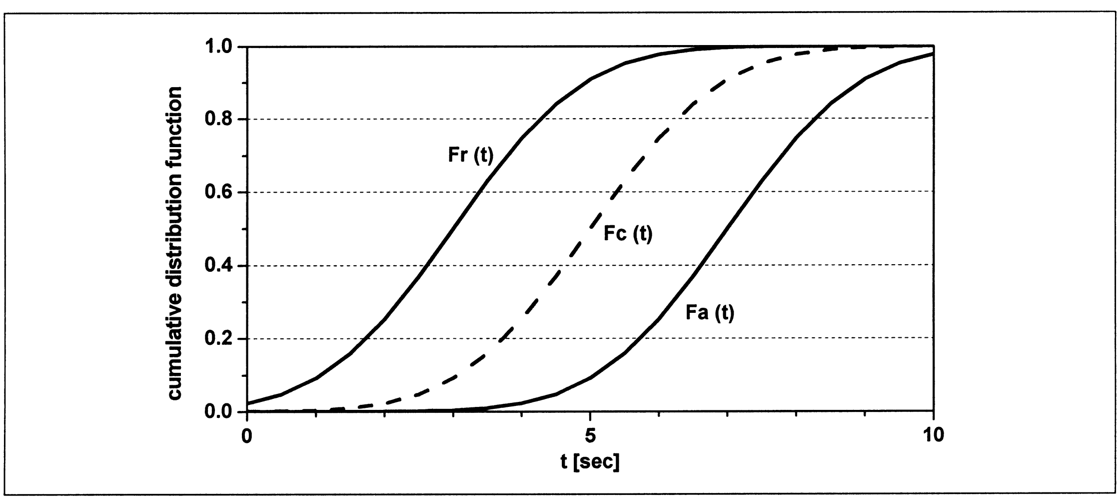

FIGURE 2 Distribution functions of accepted gaps [*Fa*(*t*)], largest rejected gaps [*Fr*(*t*)], and critical gaps [*Fc*(*t*)] within a driver population.

where:

L = maximum likelihood function
a_i = logarithm of the accepted gap of driver i
r_i = logarithm of the largest rejected gap of driver i
$F(...)$ = cumulative distribution function for the normal distribution.

The parameters μ and σ^2 are obtained by maximizing this likelihood function. In this way, the distribution of critical gaps, as well as their mean and variance, can be derived.

2.2 Practice of Determining Critical Gaps and Follow-Up Times

Measurements were taken at 30 rural intersections without traffic signals. Traffic flow at these intersections was videotaped with a time signal recorded. The time code helped identify each vehicle's arrival time at a specific point along the road within 0.04 second. Each vehicle of a minor stream was recorded in a database using vehicle type, time of arrival in front position, and time of departure. The major stream vehicles were recorded using vehicle type and time of passing a certain cross section in the intersection area. These data were then used to record drivers' follow-up times and critical gaps could be derived.

Defining arrival and departure times turned out to be difficult. Many minor stream cars approached the yield line easily and stopped right in front of the yield line. Other vehicles approached hesitantly and rolled to stop slowly. Further, the precise definition of the beginning and end of a gap between major stream vehicles was not clear: Which priority stream has to be considered to begin and end a gap? Which cross section does a vehicle of a certain major stream have to pass to begin or end a gap? To address these questions, generalizations had to be made to define starting and ending points of major stream gaps. It was expected that the results might be influenced by these definitions. Thus, after a series of experiments, the definitions were made according to the best knowledge of the research team. Once defined, however, the definitions were applied to all intersections consistently.

A computer program was developed to calculate the follow-up times and critical gaps from these data. The first results were used to verify the assumptions made for estimating the critical gaps. One question was, what type of time spaces in general could be used as the basis for determining critical gaps? A minor stream driver can use two types of time spaces: the *lag* is the time space between the arrival of the minor stream vehicle and the arrival of the first major stream car. The *gaps* are the successive time spaces between the major stream vehicles. The study team considered several ways to use the different time spaces in the analysis of critical gaps:

1. All minor stream drivers were entered into the database. The accepted and largest rejected gaps of each driver were taken from the *lag* and all *gaps* being offered to him or her. A driver who accepted the *lag* and did not reject any time gap was assigned a largest rejected time gap of 0.
2. Only drivers who rejected at least the *lag* were entered into the database. In this case the largest rejected time gap could be a *lag* or a *gap*. The accepted time gap was a *gap*.
3. Only drivers who rejected at least one *gap* were entered into the database. In this case, the largest rejected gap and the accepted gap were necessarily provided by a *gap*.

Critical gaps were determined for each of these three sample groups. The critical gaps for sample 1 turned out to be up to 1.7 seconds smaller than the critical gaps for samples 2 and 3. The critical gaps of samples 2 and 3 differed only up to 0.3 second from one another.

The results were tested by determining the capacity of some streams using Siegloch's capacity formula (Siegloch, 1973) and the three different critical gaps. The real capacity of these streams was also ascertained by counting the number of major stream vehicles and minor stream vehicles in highly frequented 1-minute intervals during times of a constant queue. Figure 3 shows a comparison of measured and calculated capacity for one stream.

The diagram and the calculation of the residual variance indicate that the capacity calculated using sample 3 best reflects real capacity. Therefore, only those minor stream drivers who rejected at least one *gap* were included in the database of drivers used to determine the critical gaps.

Another reason *lags* may contribute to the inaccuracy of the critical gaps analysis is that the *lags* were very difficult to measure. As stated previously, it was not always possible to identify the precise arrival time for a minor stream vehicle; consequently, the beginning of a *lag* was difficult to define. The ending of a *lag* also can be uncertain, depending on the geometric design of the intersection. For these reasons, this analysis did not consider the *lags* as fundamental time spaces for determining critical gaps.

The study team also questioned whether very large accepted and very small rejected gaps should be entered into the database to be used in estimating critical gaps. It can be assumed that very small gaps were rejected by all drivers and gaps larger than a specific amount were accepted by all drivers. To test this theory, the largest rejected and smallest accepted gaps of several driver populations were established as maximum and minimum margins. All drivers who accepted a gap larger than the largest rejected gap (the upper margin) were

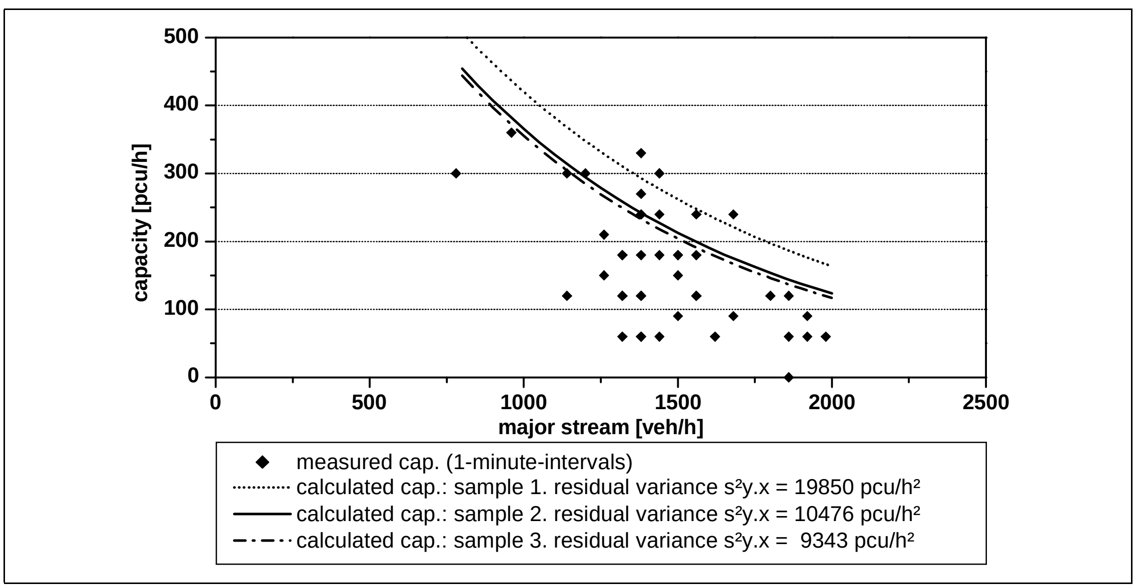

FIGURE 3 Comparison of measured capacity and three types of calculated capacities.

given an accepted gap with the value of the upper margin. All drivers with a largest rejected gap smaller than the smallest accepted gap (lower margin) were given a largest rejected gap as large as the lower margin. When the critical gaps were determined for this modified sample, it became obvious that they did not alter from the original sample. Very large and very small gaps had little influence on the resulting critical gap; therefore, all gaps could be used in determining the critical gaps, regardless of their duration.

Hypotheses regarding distribution function types for the critical gaps were also tested. As noted before, the critical gaps were assumed to follow a log-normal function. To test this assumption, an Erlang and a Weibull distribution were applied to the empirical data. The parameters of these distribution functions were estimated using the maximum likelihood technique. Figure 4 is a comparison of the three distribution types. It can be seen that optimization by the maximum likelihood technique leads to very similar distributions. Thus, the expectation for the critical gaps differs only within a very narrow margin. Also, the values of the likelihood functions are rather similar. These observations prove the consistency of the maximum likelihood technique: no matter which function was taken as a basis, the same critical gap results. Therefore, there is no reason to reject the log-normal distribution as an appropriate function to represent the critical gaps. In addition, because it is easier to use the log-normal distribution function rather than the Erlang or Weibull distribution functions, the log-normal distribution function was used to determine all critical gaps.

Finally, a very important requirement for the application of the maximum likelihood technique was examined—the consistency of the minor stream drivers. As stated before, each driver is assumed to have his own specific critical gap. This attribute is called *consistency*. A consistent driver accepts every gap larger than his critical gap and rejects

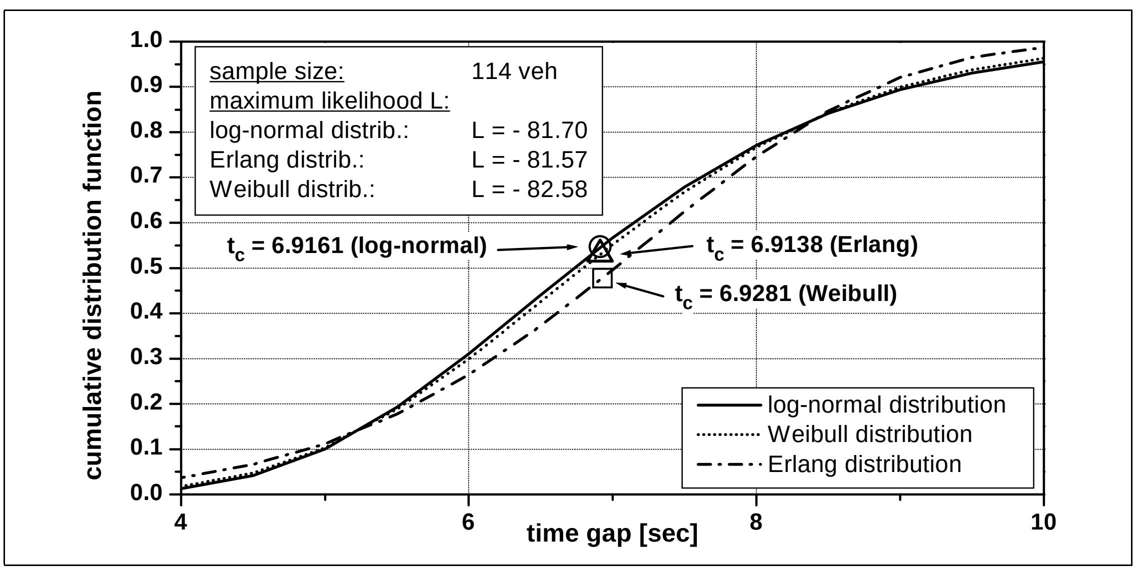

FIGURE 4 Distribution of critical gaps based on different distribution function types.

every smaller gap. The only way to recognize an inconsistent driver is when that driver accepts a gap that is smaller than a previously rejected one. Those drivers who exhibited this type of inconsistent behavior were noted in the database. Left-turning drivers from the priority road showed such behavior less often (on average 0.44%), whereas drivers crossing the major road behaved inconsistently more frequently (on average 8.46%). These inconsistencies were concentrated at rather few intersections. Specifically:

- At 85% of all examined left-turning streams from the major road, less than 1% of the driver population behaved inconsistently.
- At 15% of these left-turning streams, 1 to 5% of drivers behaved inconsistently.

The proportions were higher at the minor road crossing streams, specifically:

- At 80% of the examined crossing streams, 5 to 10% of the drivers behaved inconsistently.
- At 20% of these streams, 10 to 15% of drivers behaved inconsistently.

These values show that, although inconsistent behavior may occur among minor stream drivers, it occurs only with a minority of drivers. In general, most drivers behave consistently. Under these circumstances, use of the maximum likelihood technique is justified and the inconsistent drivers were eliminated from the database used to determine the critical gaps.

3. RESULTS

The resulting critical gaps and follow-up times varied within relatively wide margins, as illustrated in Figure 5. These variations were investigated to determine whether they are natural characteristics of the critical gaps and follow-up times or if there are any systematic influences responsible for the variations.

The critical gaps and follow-up times were tested for their dependence on several external parameters, listed in Table 1. These included discrete factors such as the type of priority rule (e.g., yield sign versus stop sign). The critical gap differences for these two samples

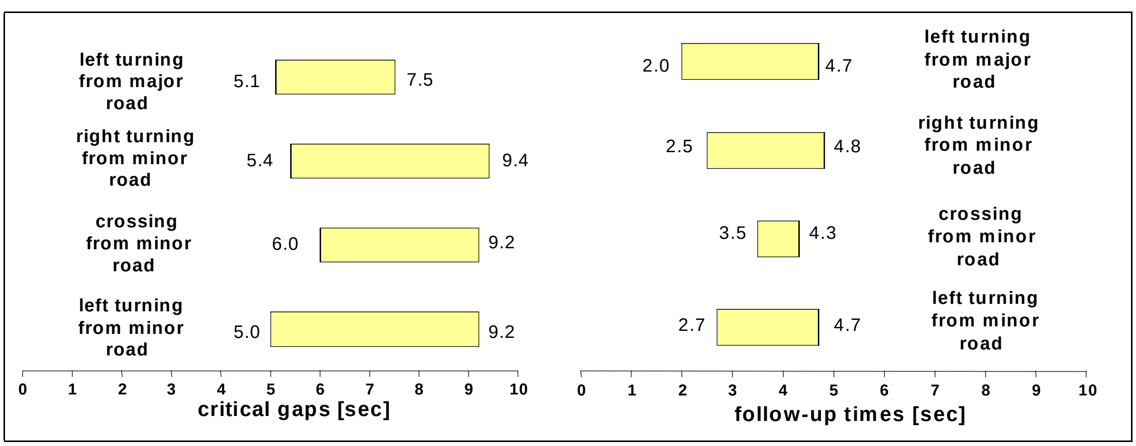

FIGURE 5 Results from the determination of critical gaps and follow-up times.

were tested using a *t*-test. Other factors such as volume of priority stream, where critical gaps and follow-up times might be related to the flow volumes during time of observation, were examined using correlation techniques. The regression coefficient was tested using a *t*-test to determine whether it was different from zero.

Table 1 shows the results of these tests. When a parameter turned out to have a significant influence on the critical gap or follow-up time it is indicated by "yes." In those cases, the

TABLE 1 Dependence of Critical Gaps (t_g) and Follow-Up Times (t_f) on External Parameters

	Left turning from major road		Right turning from minor road		Crossing from minor road		Left turning from minor road	
	t_g	t_f	t_g	t_f	t_g	t_f	t_g	t_f
Residual variances of mean critical gaps and follow-up times	0.45*	0.21*	0.95*	0.32*	0.95*	0.35*	1.14*	0.31*
Consideration of various influences:								
Yield / Stop sign			no	yes 0.19*	no	-	no	yes 0.25*
3 / 4 armed intersection	no	no	no	yes 0.26*	-	-	no	no
Rural/urban area	yes 0.35*	no	yes 0.83*	no	no	-	yes 1.01*	no
Number of lanes in minor road			no	no	yes 0.48*	no	no	no
Existence of triangular island for right turning stream from major road	yes 0.14*	yes 0.18*	no	no	no	-	no	no
Volume of major streams	yes 0.33*	no	yes 0.68*	no	no	no	yes 0.54*	yes 0.27*
Delay	no	no	no	no			no	no
Share of trucks	no	no	no	no	no	no	yes 1.03*	no
Speed on the major road	no	no	no	no	no	no	no	no
Gradient	no	no	no	no			no	no
Sight distance	no	no	no	no	no	no	no	no

In case of significant correlation (→ "yes") the residual variance is given that indicates the significance of the correlation: the smaller the residual variance is when compared to the residual variance of the mean values, the stronger the influence of the external parameter on the critical gap or follow-up time.
*Residual variance $s^2_{y.x}$ in [sec^2].

residual variance is given. From the residual variance one can see how significant the influence of the external parameter is on the critical gap or follow-up time. The smaller the residual variance is when compared to the residual variance of the mean critical gap and follow-up time, the stronger the correlation between the external parameter and the critical gap or follow-up time.

An analysis of variance (ANOVA) was performed to unite the most significant correlations to simple rules. The aim was to create formulas that allow the critical gap and follow-up time to be calculated using the most important external parameters. It should be mentioned that the influence of the major stream volumes was not used, although it existed for some of the movements. Inserting a critical gap that depends on the volume of priority stream into Siegloch's capacity formula turns this capacity formula into a function of the form:

$$C = A \cdot e^{B(q_p)^2 + D \cdot q_p} \tag{2}$$

where

C	=	capacity
A, B, D	=	parameters
q_p	=	volume of major stream.

Such a function would never result in values equal to zero, but rather would have a minimum value. However, it is unrealistic for the capacity of a minor stream to start rising again above a certain value of the major street traffic volume. It will rather fall continuously until it reaches zero.

The ANOVA led to the critical gap and follow-up time values given in Table 2. When compared to the former values determined by Harders (1976), the critical gaps for metropolitan area and the follow-up times are of approximately the same order as those given previously by Harders for urban street velocities, and the critical gaps for rural areas are a bit larger than Harders' values. The difference between the new values and Harders' values manifests itself in the influencing parameters. Harders found the speed on the major road, gradient, sight distances, and intersection angle to be the most important factors influencing the critical gaps and follow-up times. However, these correlations could not be proven with data from this investigation. The effect of speed was rejected because this correlation has not been found to exist. The other geometric effects were not relevant because all measured intersections were designed according to the guidelines and there were no extreme conditions such as acute angles between intersection arms or affected sight conditions.

TABLE 2 Critical Gaps and Follow-Up Times Recommended for Rural Unsignalized Intersections

	Critical gap t_g [sec]			
	Rural area		Metropolitan area	
	With triangular island*	Without triangular island	With triangular island	Without triangular island
Left turning from major road	6.4	5.9	6.0	5.5
Right turning from minor road	7.3		6.5	
Crossing from minor road	7.0		6.5	
Left turning from minor road	7.4		6.6	
	Follow-up time t_f [sec]			
	With triangular island*		Without triangular island*	
	Yield-sign	Stop-sign	Yield-sign	Stop-sign
Left turning from major road	2.9		2.6	
Right turning from minor road	3.1	3.7	3.1	3.7
Crossing from minor road	3.5	4.0	3.5	4.0
Left turning from minor road	3.4	3.8	3.4	3.8

*Triangular island to separate opposite right turners from opposite through traffic.

4. MODIFICATION OF THE GERMAN GUIDELINE FOR UNSIGNALIZED INTERSECTIONS

The critical gaps and follow-up times are used in the guidelines to calculate the capacity of unsignalized intersections using Siegloch's (1973) capacity formula:

$$C = \frac{3600}{t_f} \cdot e^{-q_p \cdot \left(t_c - \frac{t_f}{2}\right)} \tag{3}$$

where

C	=	capacity	(vec/h)
t_f	=	follow-up time	(sec)
t_c	=	critical gap	(sec)
q_p	=	volume of the major streams	(veh/h).

This equation reveals results similar to Harders' (1986) formula, which has been preferred for Chapter 10 of the HCM. Speaking in exact mathematical terms, Equation (3) is only valid for constant t_c and t_f values and a major stream that is exponentially distributed. Reality diverges from both assumptions. For example, non-Poisson major streams (i.e., bunched arrivals that do not have exponentially distributed major stream gaps) increase capacity as compared to Equation (3). On the other hand, distributed t_c and t_f values as they appear in reality reduce capacity to lower than those results indicated by Equation (3). However, Großmann (1991) showed that these two adverse effects that occur with realistic combinations of parameters balance one another quite well. Consequently, the use of Equation (3), which is used in the German guidelines, or Harders'formula, which is used in the HCM, is justified.

Equation (3) is represented in the German guideline by diagrams. These diagrams were established using the new critical gaps and follow-up times. Figure 6 is an example of these diagrams. Those factors designed to account for acute intersection angles or bad sight conditions, which were part of the former German guidelines (FGSV, 1991), can now be ignored.

Equation (3) or Figure 6 identifies only the potential capacity for the four possible movements at an intersection. In addition, other aspects such as the four-ranked hierarchy of priority, flaring of minor street approaches, or two-stage priority have to be taken into account to estimate movement capacities. The methodology for determining these features is not affected by the critical gaps or follow-up times. For more details of this methodology, the reader is referred to Chapter 10 of the HCM (1998) or Brilon et al. (1997).

The new critical gaps and follow-up times and the corrected German guideline were tested using the real capacity of some streams, measured as explained in Section 2.2. At the same time, the capacity of these streams was determined using Siegloch's formula and the

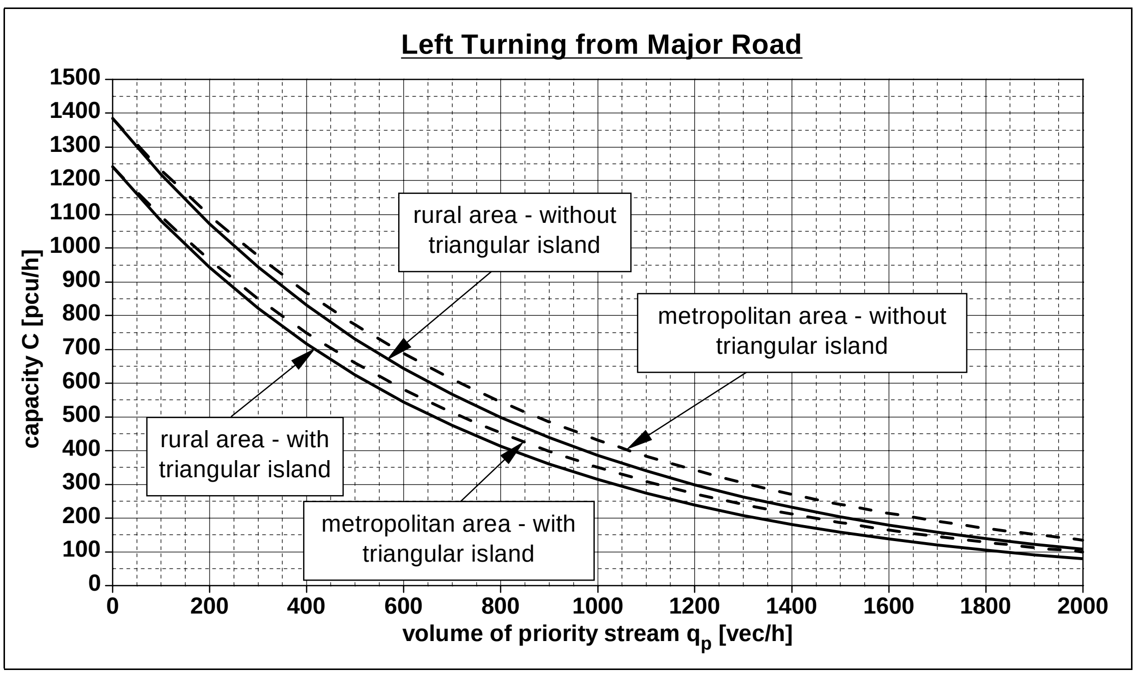

FIGURE 6 Capacity diagram with the new critical gaps and follow-up times for the example of the left turning stream from the major road.

proposed critical gaps and follow-up times. The calculated capacity represented the observed capacities quite well.

In a second step, the proposed guideline was applied to determine the quality of traffic flow estimations for several streams. This was done by calculating delay and queue length on the minor street. These parameters were also measured and compared to the calculated values. The modified capacity calculation method turned out to estimate these parameters of traffic flow quite reliably.

5. CONCLUSIONS

New critical gaps and follow-up times were determined for rural intersections in Germany. The new results differ from the former values, particularly in regard to their influencing external parameters. For the investigation, several assumptions underlying the critical gap estimation methodology have been tested. It was found that lags should not be used in the data base, either as maximum rejected or as accepted gaps. For the maximum likelihood technique, the assumed distribution function is not very influential as long as typical functions are used; consequently, use of the log-normal assumption is further justified. However, the analysis results were affected by the details of data processing, such as the exact definition of when a gap of each specific movement starts or finishes.

The investigations revealed new critical gaps and follow-up times for rural unsignalized intersections in Germany. Using limited empirical data, it could be shown that the new parameters, combined with state-of-the-art methodology for the analysis of unsignalized intersections, produced quite reliable results for both capacities and for assessment of intersection performance. These values will become part of Chapter 8 in the coming German HBS.

REFERENCES

Brilon, W., and M. Großmann. (1989). Entwicklung eines Simulationsmodells fuer Knotenpunkte ohne Lichtsignalanlagen (Development of a simulation model for intersections without traffic signals). *Forschung Strassenbau und Strassenverkehrs-technik*, Vol. 554. Bundesminister fuer Verkehr, Abt. Strassenbau, Bonn.

Brilon, W., R. Koenig, and R. Troutbeck. (1997). Useful Estimation Procedures for Critical Gaps. *Proc. Third International Symposium on Intersections Without Traffic Signals.* Portland, Oregon, U.S.A., pp. 71–87.

Brilon, W., R. Troutbeck, and M. Tracz. (1997). Review of International Practices Used to Evaluate Unsignalized Intersections. *Transportation Research Circular 468.* Transportation Research Board, Washington, D.C.

Brilon, W., and A. Weinert. (1999). Ermittlung aktueller Grenz- und Folgezeitluecken fuer Ausserortsknoten ohne Lichtsignalanlagen (Determination of actual critical gaps and follow-up times at rural unsignalized intersections). *Forschungsauftrag FE-Nr.: 02.173/1997/GGB der Bundesanstalt fuer Strassenwesen.* Bochum.

FGSV. (1991). *Merkblatt zur Berechnung der Leistungsfaehigkeit von Knotenpunkten ohne Lichtsignalanlagen* (Guideline for the evaluation of capacity of intersections without traffic signals). Koeln.

Großmann, M. (1991). Methoden zur Berechnung und Beurteilung von Leistungsfaehigkeit und Verkehrsqualitaet an Knotenpunkten ohne Lichtsignalanlagen (Methods for calculation and evaluation of capacity and traffic flow quality at intersections without traffic signals). *Schriftenreihe Lehrstuhl fuer Verkehrswesen Ruhr-Universitaet Bochum, Heft 9*, Bochum.

Harders, J. (1968). Die Leistungsfaehigkeit nicht signalgeregelter staedtischer Verkehrsknoten (Capacity of unsignalized urban intersections). *Strassenbau und Strassenverkehrstechnik,* Vol. 76. Bundesminister fuer Verkehr, Abt. Strassenbau, Bonn.

Harders, J. (1976). Grenz- und Folgezeitluecken als Grundlage fuer die Berechnung der Leistungsfaehigkeit von Landstrassen (Critical gaps and follow-up times as base for capacity calcution of rural streets). *Strassenbau und Strassenverkehrstechnik,* Vol. 216. Bundesminister fuer Verkehr, Abt. Strassenbau, Bonn.

Kyte, M., Z. Tian, Z. Mir, Z. Hameedmansoor, W. Kittelson, M. Vandehey, B. Robinson, W. Brilon, L. Bondzio, N. Wu, and R. Troutbeck. (1996*). Capacity and Level of Service at Unsignalized Intersections.* National Cooperative Highway Research Program (NCHRP) 3-46.

Siegloch, W. (1973). Die Leistungsermittlung an Knotenpunkten ohne Lichtsignalsteuerung (Capacity determination at intersections without traffic signals). *Strassenbau und Strassenverkehrstechnik*, Vol. 154. Bundesminister fuer Verkehr, Abt. Strassenbau, Bonn.

Transportation Research Board. (1998). *Special Report 209: Highway Capacity Manual*. National Research Council, Washington, D.C.

Troutbeck, R. (1992). Estimating the Critical Acceptance Gap from Traffic Movements. *Research Report 92-5*. Queensland University of Technology, Brisbane.